



A NEW STOCHASTIC AVAILABLE BIT RATE MODEL TO FIND THE AGE SPECIFIC CORONARY HEART DISEASE MORTALITY RATES USING NORMAL DISTRIBUTION

M. Vasuki* & A. Dinesh Kumar**

* Assistant Professor, Department of Mathematics, Srinivasan College of Arts & Science, Perambalur, Tamilnadu

** Assistant Professor, Department of Mathematics, Dhanalakshmi Srinivasan Engineering College, Perambalur, Tamilnadu

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Abstract:

The gender difference (gender gap) in mortality due to coronary heart disease (CHD) decreases with age. This relationship has not been well characterized in diverse populations. The purpose of this study was to examine the age specific coronary heart disease mortality rates in men compared with women through multi server queueing model (ABR Model) with the help of normal distribution.

Key Words: Coronary Heart Disease, Hormone Replacement Therapy, Available Bit Rate Model & Normal Distribution.

Introduction:

Although Coronary Heart Disease (CHD) is the leading cause of death in men and women, age-specific CHD mortality rates are strikingly higher in men compared with women. In general, both CHD incidence and mortality rates in women lag 10 years behind those of men [9]. It is well established that the gender difference is more pronounced at younger ages, such that 1 in 17 women has had a coronary event before age 60, in contrast with 1 in 5 men. The gender difference has been reported to decrease with age and after age 60, CHD accounts for 1 in 4 deaths in both sexes. Previous United States based studies addressing gender differences in CHD mortality have been limited to predominantly Caucasian populations [7, 22 & 25]. Many studies have also examined black and white differences in CHD mortality, but none has directly compared the gender gap between ethnic groups [2, 6, 8, 26 & 27]. The gender gap in CHD mortality has been attributed to various factors. Differential prevalence and impact of traditional cardiovascular risk factors have been shown to account for part but not all of the gender difference. Estrogen has been implicated as a possible protective factor in women because of an observed 2-fold increased CHD incidence in surgically postmenopausal vs premenopausal women of the same age. However, the use of hormone replacement therapy (HRT) has not been shown to reduce CHD events in postmenopausal women and the role of endogenous estrogen in the cardio protection of women compared with men is not completely understood. International data suggest that geography, secular trends, and environmental factors also play a role in gender differences in CHD occurrence [26]. The purpose of this study was to examine the age specific coronary heart disease mortality rates in men compared with women through multi server queueing model (ABR Model) with the help of normal distribution.

The diverse characteristics and service requirements of the different traffic types that are carried by ATM (Asynchronous Transfer Mode) networks have led to the definition of different service categories that should be offered to users of such a network. We briefly discuss those differences, distinguishing three large categories: Constant Bit Rate (CBR) traffic, Variable Bit Rate (VBR) traffic and Available Bit Rate (ABR) traffic. The CBR service class guarantees a fixed pre-determined transmission capacity to its users. Therefore, this service is useful for traffic that requires both very small (or no) delays and very small (or no) losses. At the burst level (where we distinguish different bursts of traffic coming from the same connection, but not the separate ATM cells that form a burst), it is reasonable to assume that all CBR traffic requires a fixed amount of capacity over time. In all further considerations, we will leave out the CBR traffic and use the term "capacity" to indicate the total capacity minus the capacity reserved for CBR traffic.

For VBR traffic, we make a subdivision into real time and non real time connections. For both these subclasses, the users must specify many characterizing parameters such as minimum cell rate, mean cell rate, peak cell rate and maximum burst size. The difference lies in the requirements. The main issue for real time connections such as voice and possibly video is the delay of the transmission; the loss of small amounts of information during the transmission is less important for these connections. This traffic lends itself very well for multiplexing. On the other hand, non real time VBR traffic requires small losses and the delays are less important. To ensure that losses are small, large buffers are used to store non real time VBR traffic when the communication network is heavily loaded.

The last category, ABR traffic, was introduced to cope with specific problems that arise when transmitting data. For this traffic, losses lead to retransmission of data (because of the extreme sensitivity to losses), which introduces a lot of overhead in implementations. Since transmission delays are of less importance for data traffic, the setting of non real time VBR seems to be the appropriate one to carry data traffic. However,

data traffic is very bursty and the required parameters for VBR connections are difficult to specify by the users. For ABR connections, no parameters need to be specified. Only a small amount of capacity is reserved for the transmission of ABR traffic. Additionally, the capacity that is not currently being required by VBR (and CBR) traffic is used for ABR traffic. When the total capacity currently available to ABR is too small, ABR traffic is stored in very large buffers, ensuring a small loss probability, until the available capacity increases again. The advantage here is that ABR traffic gets all the capacity that is left over. For the server, this means a higher utilization of the network's resources. As pointed out above, the main service guarantee for ABR traffic is a very small loss fraction or, in principle, no loss at all. No guarantee can be given on transmission delays.

Model Classification:

A special issue of ABR is that the available capacity should be shared fairly among all ABR users. In queueing models, it seems reasonable to incorporate this with the queue discipline of processor sharing. In this discipline, all "customers" receive an equal share of the service capacity. In addition to the large storage buffers, some feedback control mechanism can be used to keep the loss of information small. The buffers can store incoming data that cannot be transmitted immediately, due to a temporarily overloaded system. Feedback control can be used to slow down data sources when the buffers are heavily loaded and an overflow may occur. We refer to the ATM Forum for more detailed specifications of ABR.

In [18], Ritter investigates the problem of dimensioning the buffer for ABR traffic in order to avoid large losses. In [19] and [20], Ritter considers the case with feedback control, under the assumption that the source of ABR traffic is saturated. A drawback in most studies is the assumption of a fixed available capacity for transmission of ABR traffic. As it was pointed out above, one of the essential features of ABR is that it makes use of the capacity that is left over by VBR traffic. Therefore, there is a need for a detailed performance analysis of ABR in the presence of other traffic.

Our model is basically a multi-server queue with two types of customers: (i) high priority customers (real time VBR traffic) and (ii) low priority customers (ABR traffic). We assume that the high priority customers have a waiting room with a finite, typically small capacity - thus modeling the real time requirement and each accepted customer is served by a single server. Low priority customers have an infinite waiting room and equally share the remaining capacity according to the processor sharing principle - this models the large storage buffers for ABR traffic and the fair sharing of the available capacity between ABR users. The servers at the service station are divided into two groups: (i) there are N servers that are dedicated principally to the high priority customers and (ii) N_L servers are purely reserved for the low priority traffic. On the normal servers, the high priority customers have preemptive priority over the low priority customers.

We point out that this is a call level model: A customer represents a request of an ABR source to transmit data, and the service requirement of the customer is identified with the amount of data to be transmitted. In our analysis, we assume that arrivals occur according to two independent Poisson processes. This assumption is justified in the case where many sources are connected to the communication network. Although we present the model in the context of ABR traffic, it can just as easily be seen in the context of existing situations, where real time VBR has priority over non real time VBR. In this case, the processor sharing discipline for the low priority traffic should be replaced by the First Come First Served (FCFS) discipline. Also, the processor sharing among ABR sources is interesting in the light of per Virtual Connection (VC) queueing, where sources do not queue behind one another, but each source gets a separate access to the server. The feature of processor sharing can further be generalized to weighted fair queueing, where the total capacity is divided between the active sources according to some weighting factors.

Related two dimensional Markov models have been studied in a number of papers. The case where both types of customers have an infinite waiting space, and within each customer type the service discipline is FCFS [11]. In [3] treated the preemptive case with processor sharing among the low priority customers. A discrete time variant modeled as an M/G/1 type Markov Chain is considered in [4]. A more extensive treatment of the spectral analysis of M/G/1 type Markov Chains is given in [5]. A model related to the one presented in this paper, addressing the case with finite buffer capacity, is treated in [16]. In [1], consider a model similar to ours and give various performance measures in terms of the steady state distribution, rather than analyzing this distribution in greater detail. Our main goal is to give a detailed analysis of the steady state distribution itself.

In our analysis we are inspired, but we make use of methods from other approaches. Instead of transforming the involved distributions into generating functions, the present work focuses directly on the distribution itself. It does so relying mainly on the matrix geometric approach of [15] and the spectral expansion approach [10] & [12]. In this paper, we give a complete characterization of the joint distribution of the numbers of customers of both types in the system at steady state. Numerical results are presented to illustrate the effect of high priority traffic on the service performance of low priority traffic.

4. The Equilibrium Distribution:

Consider R has $N + K + 1$ different eigenvalues in the interval $(0,1)$; therefore, the equilibrium distribution can be written as

$$\bar{\pi}_j = \sum_{k=0}^{N+K} \alpha_k (r_k)^j \bar{v}_k, \quad j \geq 0$$

We order the eigenvalues of R as $0 < r_0 < r_1 < \dots < r_{N+K} < 1$ (note that r_k is the root of $r_k(z)$ in the unit interval), and construct the diagonal matrix $\Lambda = \text{dia}[r_0, r_1, \dots, r_{N+K}]$. The corresponding eigenvectors $\bar{v}_0, \bar{v}_1, \dots, \bar{v}_{N+K}$ compose the matrix V , $\bar{v}_k$ being the $(k+1)$ row of V . Remember that $\bar{v}_k$ can be found as the left nullvector of the matrix $T(r_k)$. We have the Jordan decomposition $R = V^{-1}\Lambda V$. The equilibrium distribution is fully determined as soon as we have $\bar{\pi}_0$, which must satisfy:

$$\bar{\pi}_0 [Q_{00} + RT^{(-)}] = \bar{0} \quad (1)$$

Here $Q_{00} + RT^{(-)}$ is an irreducible generator, and therefore (1) has a positive solution, which is unique up to multiplication by a scalar. Obviously, if we let e be the $(N+K+1)$ dimensional vector with all elements equal to 1, it must be that:

$$\bar{\pi}_0 (I - R)^{-1} e = \bar{\pi}_0 \sum_{j=0}^{\infty} R^j e = \sum_{j=0}^{\infty} \bar{\pi}_j e = 1 \quad (2)$$

Together, (1) and (2) completely determine $\bar{\pi}_0$. Since we want to have the $\bar{\pi}_k$ as in (8), or equivalently in matrix form

$$\bar{\pi}_j = \bar{\alpha} \Lambda^j V$$

We write (1) and (2) to:

$$\begin{aligned} \bar{\alpha} [V Q_{00} + \Lambda V T^{(-)}] &= \bar{0} \\ \bar{\alpha} (I - \Lambda)^{-1} V e &= 1 \end{aligned}$$

This determines $\bar{\alpha} = (\alpha_0, \alpha_1, \dots, \alpha_{N+K})$

An alternative way of finding the coefficients α_k in the present model is by using $p_i = p_0((\rho_H)^i / i!)$, $i = 1, 2, \dots, N-1$. Denoting by $\bar{p}$ the vector $(p_0, p^1, \dots, p_{N+K})$ with $p_i = P\{X_H = i\}$, it must hold that:

$$\bar{\alpha} (I - \Lambda)^{-1} V = \sum_{j=0}^{\infty} \bar{\pi}_j = \bar{p}$$

In particular, the low priority queue length distribution is given by:

$$P\{X_L = j\} = \bar{\alpha} \Lambda^j V e = \sum_{k=0}^{N+K} \alpha_k (r_k)^j \bar{v}_k e$$

If we had used the normalization $\bar{v}_k e = 1$ for the eigenvectors, this would have become

$$P\{X_L = j\} = \sum_{k=0}^{N+K} \alpha_k (r_k)^j \quad (3)$$

However, note that it remains to be verified whether the elements of some $\bar{v}_k$ sum up to 0. If that is the case, the corresponding term in (3) vanishes.

5. Numerical Results:

In this section, we present some numerical results to illustrate the influence of varying server availability on the performance of low priority traffic. For normalization purposes we choose $\mu_L = 1$, and in all cases we take $N = 17$. Further, we choose the extreme cases where there is no waiting room for high priority customers ($K = 0$), and no reserved capacity for low priority customers $N_L = 0$. Before discussing the numerical experiments, we first make some intuitive remarks about the cases when λ_H and μ_H are very large or very small compared to λ_L and μ_L . These intuitions can be proved formally. First, we fix λ_L, μ_L and ρ_H and let μ_H (or equivalently λ_H) go to infinity. Note that with fixed ρ_H , the mean number of servers available to the low priority customers, $N - E[X_H]$ is also fixed. As $\mu_H \rightarrow \infty$, low priority customers are in the system so long, that the mean number of available servers during the sojourn time of a low priority customer will be close to the mean number of available servers in steady state (i.e., close to $N - E[X_H]$). Therefore, it is to be expected that the low priority traffic in the limit (as $\mu \rightarrow \infty$) experiences the system as if it were an M/M/1 processor sharing queue with server capacity $c = N - E[X_H]$. For the queue length distribution, this model coincides with that of the regular M/M/1 queue with traffic load ρ_L / c .

On the other hand, if we let $\mu_H \rightarrow 0$, (again for fixed λ_L, μ_L and ρ_H), the opposite happens: the number of servers available to low priority customers changes very slowly compared to their sojourn times. An arriving low priority customer finding no available server must wait until one becomes available before receiving any service. The mean of this waiting time is $1 / N\mu_H$ and tends to infinity as $\mu_H \rightarrow 0$. Since the probability of finding all servers occupied is positive, the expected sojourn time of the low priority customers also goes to infinity, and by Little's law, so does $E[X_L]$. In our experiments, we are interested in the behavior of the mean and variance of the number of low priority customers in the system, at some fixed system load $\rho := \rho_L + E[X_H]$. Therefore, for different values of μ_H and with μ_L normalized to 1, we vary λ_L and λ_H , keeping ρ constant.

Now we have chosen $\rho = \frac{7}{10} N$. We consider three values for μ_H : $\mu_H = \frac{1}{5}, 1$, and 5. Further, we also plot the results for the M/M/1 queue with server capacity $c = N - E[X_H]$. We have already argued that this model corresponds to the case $\mu_H = \infty$. Therefore, in this case:

$$P\{X_L = j\} = \left(1 - \frac{\rho_L}{c}\right) \left(\frac{\rho_L}{c}\right)^j, j = 0, 1, 2, \dots \quad (4)$$

Since $\mu_L = 1$, in the experiments we vary λ_L from 0 to $\frac{7}{10} N$. At the same time, λ_H decreases such that at all times $\rho = \rho_L + E[X_H] = \frac{7}{10} N$. The mean $E[X_L]$ and variance $\text{var}[X_L]$ of the number of low priority customers

are plotted, respectively. On the horizontal axis λ_L is normalized to ρ_L/ρ . We see that $E[X_L]$ and $var[X_L]$ are particularly sensitive to μ_H when ρ_L and $E[X_H]$ are of the same order.

$$E[X_L] = \frac{\rho_L/c}{1-\rho_L/c} = \frac{\rho_L}{N-E[X_H]-\rho_L} = \frac{\rho_L}{N-\rho}$$

$$var[X_L] = \frac{\rho_L/c}{1-\rho_L/c} \left(1 + \frac{\rho_L/c}{1-\rho_L/c} \right) = \frac{\rho_L}{N-\rho} \left(1 + \frac{\rho_L}{N-\rho} \right) \quad (5)$$

4. Example:

To examine the gender gap in CHD mortality across age groups and to compare the age dependency of the gender gap between blacks and whites, we conducted a prospective cohort study combining data from 9 United States epidemiological studies (Atherosclerosis Risk in Communities Study, Charleston Heart Study, Evans County Study, Framingham Heart Study [original and offspring cohorts], National Health Examination Follow-up Study, Rancho Bernardo Study, San Antonio Heart Study, and Tecumseh Community Health Study). Baseline examinations were performed between 1958 and 1990 (depending on the study), and mean follow up was 13.7 years in general communities in several United States geographic areas. We included 39,614 subjects >30 years and free of cardiovascular disease (CVD) at baseline (18% blacks, 37% men). Completion of follow-up was >97% for all studies. As the main outcome measures, age specific CHD mortality rates and male/female CHD mortality hazard ratios were calculated using Cox hazards regression [13 - 14, 17 & 23 - 24].

Figure (1): Age Specific CHD mortality rates in men compared with women. The horizontal lines illustrate the lag times of CHD mortality rates between men and women.

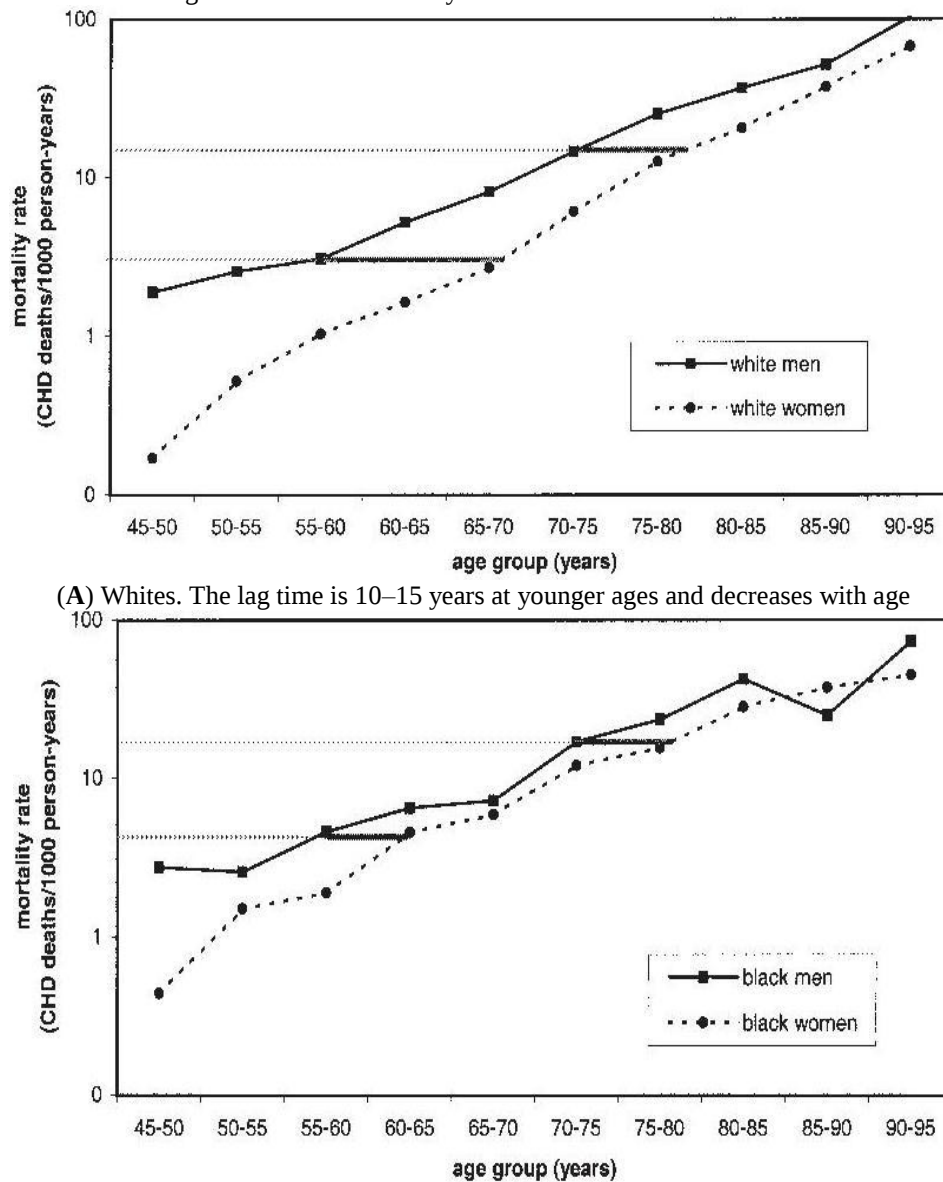
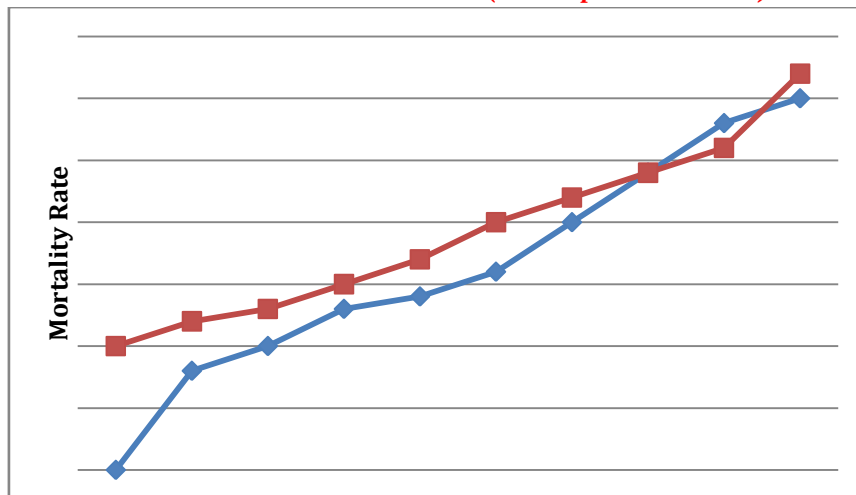
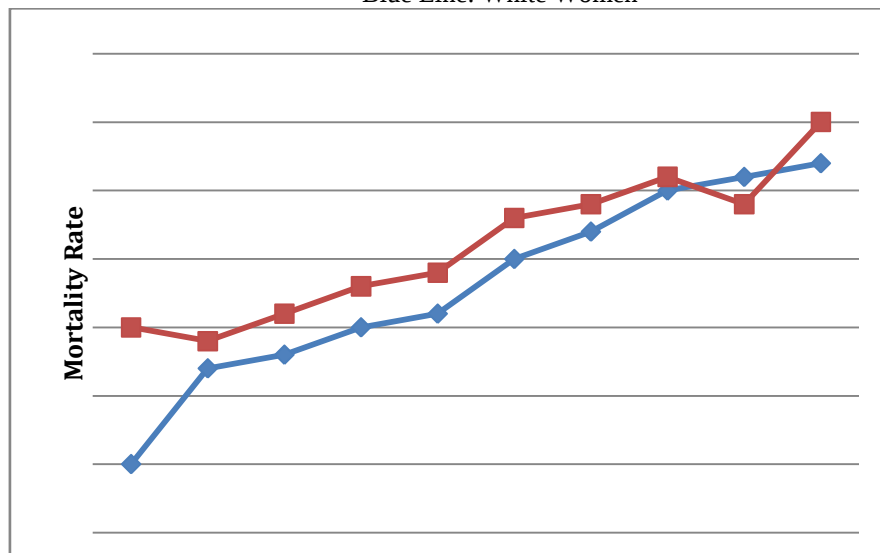


Figure (2): Age Specific CHD mortality rates in men compared with women. The horizontal lines illustrate the lag times of CHD mortality rates between men and women using Normal Distribution.



(A) Whites. The lag time is 10–15 years at younger ages and decreases with age
 Red Line: White Men
 Blue Line: White Women



(B) Blacks. The lag time is 5–10 years
 Red Line: Black Men
 Blue Line: Black Women

5. Conclusion:

The gender difference in coronary heart disease mortality was more pronounced in whites than in blacks at younger ages. This discrepancy was not explained by adjustment for coronary heart disease risk factors and suggests that other factors may be responsible for the ethnic variation in the gender gap. By using normal distribution the mathematical model gives the result as same as the medical report. The medical reports {Figure (1)} are beautifully fitted with the mathematical model {Figure (2)}; (*i.e*) the results coincide with the mathematical and medical report.

6. References:

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